## Section 6.2
## Antidifferentiation by Substitution

### Definite Integral vs. Indefinite Integral

Definite Integral is a number, the limit of a sequence of Riemann Sums

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Indefinite Integral is a family of functions having a common derivative.

$$\int f(x)\,dx = F(x) + C$$

# Example

$$\int x^2 - \sin x \, dx$$

# General Antiderivative - Indefinite Integral

$$\int x^2 \, dx =$$

$$\int \frac{1}{x + 1} \, dx =$$

$$\int \frac{1}{x^3} \, dx =$$

$$\int \cos(2x)\, dx =$$

$$\int e^{3x}\, dx =$$

$$\int x^2 + 7\, dx =$$

$$\int \sin(2x)\,dx =$$

$$\int e^{(1/3)x}\,dx =$$

## Properties of Indefinite Integrals

$$\int k\, f(x)dx = k\int f(x)\, dx \quad \text{for any constant } k$$

$$\int (f(x) \pm g(x))\, dx = \int f(x)\, dx \pm \int g(x)\, dx$$

## Power Formulas

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C \text{ when } n \neq -1$$

$$\int u^{-1} \, du = \int \frac{1}{u} \, du = \ln|u| + C$$

## Trigonometric Formulas

$$\int \cos u \, du = \sin u + C$$

$$\int \sin u \, du = -\cos u + C$$

$$\int \sec^2 u \, du = \tan u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

## Exponential and Logarithmic Formulas

$$\int e^u \, du = e^u + C$$

$$\int a^u \, du = \frac{a^u}{\ln a} + C$$

$$\int \ln u \, du = u \ln u - u + C \quad \text{(See Example 2)}$$

$$\int \log_a u \, du = \int \frac{\ln u}{\ln a} \, du = \frac{u \ln u - u}{\ln a} + C$$

## More Examples

$$\int u^{-1}\, du =$$

$$\int \ln u\, du =$$

**Let $f(x) = x^3 + 1$ and let $u = x^2$. Find each of the following antiderivatives in terms of x:**

**a.)** $\int f(x)\, dx =$

**b.)** $\int f(u)\, du =$

**c.)** $\int f(u)\, dx =$

$$\int \cos(x^3)\,dx =$$

# Substitution in Indefinite Integrals

$$\int \cos(x^3) * x^2 \, dx =$$

$$\int \tan x \, dx =$$

$$\int 2x\sqrt{4 - x^2}\,dx =$$

$$\int \frac{4e^x}{\sqrt{e^x}}\,dx =$$

$$\int \sin x \, e^{\cos x} \, dx =$$

$$\int x^2 \sqrt{5 + 2x^3}\, dx =$$

$$\int \cot 7x \, dx =$$

**Find the indefinite integrals. In each case you can use a trigonometric identity to set up a substitution.**

$$\int \frac{dx}{\cos^2 2x} =$$

$$\int \cot^2 3x \, dx =$$

$$\int \cos^3 x \, dx =$$

# Substitution in Definite Integrals

**Evaluate** $$\int_{0}^{\pi/3} \tan x \sec^2 x \, dx =$$

$$\int_{0}^{\pi} e^{\sin x} \cos x \, dx =$$

Evaluate $\displaystyle\int \frac{t}{2t^2 + 1}\, dt = \frac{1}{4}\int \frac{1}{u}\, du$

$= \frac{1}{4} \ln|2t^2+1| + C$

$u = 2t^2 + 1$

$\frac{du}{dt} = 4t$

$du = 4t\, dt$

$\frac{du}{4} = t\, dt$

$$\int \frac{\ln x}{x}\, dx =$$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$$\int u\, du = \frac{u^2}{2} + C$$

$$= \frac{(\ln x)^2}{2} + C$$

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt =$$

$u = \cos(2t+1)$

$du = -2\sin(2t+1)\,dt$

$$= -\frac{1}{2}\int \frac{1}{u^2}\,du$$

$$= -\frac{1}{2}\int u^{-2}\,du = -\frac{1}{2}\left(\frac{u^{-1}}{-1}\right) + C$$

$$= \frac{1}{2}\cdot\frac{1}{\cos(2t+1)} + C$$

$\frac{1}{2}\sec(2t+1) + C$

$$\int_2^5 \frac{\sqrt{\ln x}}{x}\,dx = \int_2^5 \sqrt{u}\,du$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \int_2^5 u^{\frac{1}{2}}\,du$$

$$= \frac{2u^{\frac{3}{2}}}{3}\Bigg|_2^5$$

$$= \frac{2\ln x^{\frac{3}{2}}}{3}\Bigg|_2^5$$

**Evaluate** $\displaystyle\int_0^1 \frac{x}{x^2 - 4}\,dx =$

$u = x^2 - 4$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int_0^1 \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u|\Big|_0^1$

$= \frac{1}{2}\ln|x^2-4|\Big|_0^1$

$= \frac{1}{2}\ln|1^2-4| - \frac{1}{2}\ln|0-4|$

$= \frac{1}{2}\ln 3 - \frac{1}{2}\ln 4$

$= \ln\sqrt{3} - \ln\sqrt{4}$

$= \ln\frac{\sqrt{3}}{2}$